# **3D** Kinematics Control for a Hyper Redundant Robot

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**Abstract:** This paper presents the simulation, implementation and control problem for a class of hyper redundant manipulators – the tronconic tentacle arms. A tentacle robot changes its configuration by bending a continuous backbone formed of sections connected in a serial configuration. Such tentacle arm has a variable length and theoretically it can achieve any position and orientation in 3D space. A tentacle arm prototype was designed and the practical realization is now running. A solution for kinematics of this arm is presented.

Keywords: robot arm, tentacle, robot control, modeling.

## 1. INTRODUCTION

A tentacle manipulator is a hyper redundant or hyper degree of freedom manipulator and lately can be observed a rapidly expanding interest in their study and construction. The control of these systems is very complex and a large number of researchers have tried to offer solutions for this difficult problem. Various solution have been tried in order to obtain a fast and accurate control algorithm (Cieslak and Morecki, 1999; Hirose, 1993; Immega and Antonelli, 1995).

The research group from the University of Craiova, Romania started working in the field of hyper redundant robots over 20 years ago. Since 2008, the research group designed a new experimental platform for tentacle manipulators. First, a cylindrical structured was designed and experimented. A new prototype based on truncated cone segments was designed and implemented.

This new robot is actuated by stepper motors. The rotation of these motors generates the actuation for cables which, by correlated screwing and unscrewing of their ends, determines their shortening or prolonging. By consequence, the tentacle curvature changes with the cables length variation. The backbone of the tentacle is an elastic rod made out of steel, which sustains the entire structure and allows the bending. Depending on which cable shortens or prolongs, the tentacle bends in different planes, each one making different angles (rotations) respective to the initial coordinates frame attached to the manipulator segment – i.e. allowing the movement in 3D (Blessing and Walker, 2004). The final goal is to obtain a closed loop structure control, based on the information

given by a machine vision system which is able to offer the real 3D positions and orientations of the tentacle segments (Ivănescu *et al.*, 2006).

The designed and implemented tentacle arm prototype is shown in Fig. 1. In this paper, the inverse kinematics problem is reduced to determining the time varying backbone curve behavior (Chirikjian 1995; Gravagne and Walker, 2000; Walker and Carreras, 2006).

First, the model of the tentacle is presented in order to obtain a shape description. Second, a model for the curvature of one segment is discussed. Following, the pseudo real time actuation solution is presented. Results for the simulation are plotted in the fifth chapter.



Fig. 1. A tentacle arm prototype

## 2. MODEL OF TENTACLE ARM

#### 2.1 Problem formulation.

In order to control a hyper-redundant robot we have to develop a method to compute the positions for each one of its segments (Boccolato et al., 2009). By consequence, given a desired curvature S\*(x, t<sub>f</sub>) as sequence of semi circles, the goal is to identify how to move the structure in order to obtain s(x, t) such that

$$\lim_{t \to t_f} s(x,t) S^*(x,t_f) \tag{1}$$

where x is the column vector of the shape description and  $t_f$  is the final time (see Fig. 2).



Fig. 2. The description of the desired shape

## 2.2 Tentacle shape description.

For describing the tentacle's shape we will consider two angles ( $\alpha$ ,  $\theta$ ) for each segment, where  $\theta$  is the rotation angle around Z-axis and  $\alpha$  is the rotation angle around the Y-axis (Fig. 2). For describing the movement we can use the roto-translation matrix considering  $\theta = 2\beta$  as shown in Fig. 3.



Fig. 3. The 3D curvature parameters

Fig. 4. shows the relation between the orientation of one segment and the curvature angle  $\theta$  of the previous segment.



Fig. 4. Curvature and relation between  $\theta$  and  $\alpha$ .

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The generic matrix in bidimensional space system that expresses the coordinates of the next segment related to the previous reference system can be written as follow:

$$\begin{bmatrix} \cos(2\beta) & \sin(2\beta) & L \cdot \sin(\beta) \\ -\sin(2\beta) & \cos(2\beta) & L \cdot \cos(\beta) \\ 0 & 0 & 1 \end{bmatrix}$$
(2)

 $\langle \rangle$ 

In 3D space we cannot write immediately the dependence that exists between two segments. This relation can be obtained through the pre-multiplication of generic rototranslation matrix. One of the possible combinations to express the coordinate of the next segment related to the frame coordinate of the previous segment is the following:

$$R_{generic}^{i} = R_{z}^{i}\left(\theta^{i}\right) \cdot Tr_{y}\left(V^{i}\right) \cdot R_{y}^{i}\left(\alpha^{i}\right) \cdot R_{z}^{i}\left(\theta^{i}\right)$$
(3)

where  $R_z^i(\theta^i)$  and  $R_y^i(\alpha^i)$  are the fundamental rototranslation matrix having 4x4 elements in 3-D space, and  $Tr_v(V)$  is a 4x4 elements matrix of pure translation in 3-D space and where V<sub>i</sub> is the vector describing the translation between two segments expressed in coordinates of i-th reference system.

The main problem is how to obtain an imposed shape for the tentacle arm. In order to control the robot, we need to obtain the relation between the lengths of the wires and the shape of the segment.

#### 3. CURVATURE OF ONE SEGMENT

In the current stage of our research, a decoupled approach is used for the robot control scheme. Thus the three segments are controlled separately, without considering the interaction between them. This section presents the direct kinematics for one segment of the tentacle.

### 3.1 Direct kinematic of the wires

The geometry of one segment for the plane case is described in Fig. 5. The curvature's angle  $\theta$  of the segment is considered as the input parameter, while the lengths  $L_1$  and  $L_2$  of the control wires are the outputs.



Fig. 5. The geometry of one segment.

The radius R of the segment curvature is obtained using equation (4):

$$R = \frac{H}{\theta} \tag{4}$$

where H is the height of the segment. The following lengths are obtained from Fig. 5, based on the segment curvature:

$$L_{11} = \overline{CP_4} = R + \frac{D_1}{2} \quad L_{12} = \overline{CP_1} = R + \frac{D_2}{2}$$

$$L_{21} = \overline{CP_3} = R - \frac{D_1}{2} \quad L_{22} = \overline{CP_2} = R - \frac{D_2}{2}$$
(5)

where  $D_1$  and  $D_2$  are the diameters of the segment end discs.

Based on the Carnot theorem, the lengths  $A_1$  and  $A_2$  are then obtained:

$$A_{1} = \sqrt{L_{11}^{2} + L_{12}^{2} - 2 \cdot L_{11}^{2} \cdot L_{12}^{2} \cdot \cos\theta}$$

$$A_{2} = \sqrt{L_{21}^{2} + L_{22}^{2} - 2 \cdot L_{21}^{2} \cdot L_{22}^{2} \cdot \cos\theta}$$
(6)

The control wires curvature radius R1 and R2 are given by the relations (7):

$$R_1 = \frac{A_1}{2 \cdot \sin\frac{\theta}{2}} \quad R_2 = \frac{A_2}{2 \cdot \sin\frac{\theta}{2}} \tag{7}$$

Finally, the lengths of the control wires are obtained as in (8):

$$L_{w1} = R_1 \cdot \theta = \frac{A_1 \cdot \theta}{2 \cdot \sin \frac{\theta}{2}}$$

$$L_{w2} = R_2 \cdot \theta = \frac{A_2 \cdot \theta}{2 \cdot \sin \frac{\theta}{2}}$$
(8)

For the 3D case, a virtual wire is considered, which gives the  $\alpha$  direction of the curvature. Considering one virtual wire in the direction of the desired curvature having length calculated as below.

Firstly the following lengths are computed:

$$L_{11} = R + \frac{D_1}{2} \cdot \cos(\alpha_1) \qquad L_{12} = R + \frac{D_2}{2} \cdot \cos(\alpha_1)$$

$$L_{21} = R + \frac{D_1}{2} \cdot \cos(\alpha_2) \qquad L_{22} = R + \frac{D_2}{2} \cdot \cos(\alpha_2)$$

$$L_{31} = R + \frac{D_1}{2} \cdot \cos(\alpha_3) \qquad L_{22} = R + \frac{D_2}{2} \cdot \cos(\alpha_3)$$
(9)

where  $\alpha_n$  is according to Fig. 6:

$$\begin{cases} \alpha_1 = -\alpha \\ \alpha_2 = 120^\circ - \alpha \\ \alpha_3 = 240^\circ - \alpha \end{cases}$$
(10)

Based on (6) and (9) the curvature radiuses  $R_1$ ,  $R_2$  and R3 of the three control wires are then obtained. Finally the lengths of the control wires are computed with (11):

$$L_{w1} = R_1 \cdot \theta$$

$$L_{w2} = R_2 \cdot \theta$$

$$L_{w2} = R_2 \cdot \theta$$
(11)



Fig. 6. Projection of the wire to determine the  $\alpha$  direction

Apart from the system presented we can obtain two useful relations:

$$\begin{cases} \sum_{i=1}^{3} \cos(\alpha_{i}) = 0 \\ \frac{1}{3} \sum_{i=1}^{3} Lw_{i} = L \end{cases}$$
(12)

The second equation of (12), can be utilized to estimate the virtual compression or the extension of the central bone. We call that virtual compression because before we compress the central bone, the robot will twist to find the shape determined by the erroneous length of the wires.

## 4. TENTACLE ARM ACTUATION PROBLEM

#### 4.1 Pseudo real time system. The problem.

The actual implementation of the tentacle robot is not equipped with a real time control system, as showed in Fig. 10. This means that the speed cannot be changed on the fly during a movement. The solution to avoid the mandatory of using a real time system is to impose the wires' speeds constant. As we can see from the equations (13), (14) and (15), the relations are not linear and time dependent.

$$R(t) = \frac{\overline{L}_{CB}}{\theta(t)}$$
(13)

$$\dot{L}_{wn} = \frac{d}{dt} \left[ \left( \frac{\overline{L}_{CB}}{\theta(t)} - \Delta R \cdot \cos(\alpha) \right) \cdot \theta(t) \right] = \text{Constant}$$
(14)

$$\dot{L}_{wn} = \frac{d}{dt} \left[ \overline{L}_{CB} - \Delta R \cdot \cos(\alpha) \cdot \theta(t) \right] = \text{Constant}$$
(15)

The problem, which we focus our attention on, is that of avoiding the deformation of the structure. So we would like to follow a path with the robot in the joint space in order to obtain the wanted shape without introducing vibration and moving with constant speed the wires.

#### 4.2 The solution

It is not possible to follow every trajectory in the state space, maintaining the wires' constant speed. We can consider the trajectory in the state space divided in small steps. Between the steps, we need to avoid the structure's vibration without taking into consideration the trajectory.

Considering the small steps, for a typical movement of a 3 links robot we consider two points in the joint space X(t) as follows:

- the starting shape of the little step (in radians) at time  $t_0$  (in seconds):

$$\left[q_1^0, q_2^0, q_3^0\right] = X^0 \tag{16}$$

- the final shape at time  $t_f$  (in seconds):

$$\left[q_{1}^{f}, q_{2}^{f}, q_{3}^{f},\right] = X^{f}$$
(17)

We also consider that  $q_n^i = (\alpha_n^i, \theta_n^i)$  are the joint variable of the n-th segment (in radians) at the time *i* (in seconds).

*Hypothesis 1:* We impose the constant speed of the motors during the movement in the small steps.

The vibrations in the structure are given by the structure twisting; this twisting is given from the incompatible lengths of the three wires of the segment. In order to avoid the structure vibration we want to move the robot from the starting position to the final position maintaining constant speeds of the wires and maintaining valid (12). We can analyze the direct kinematic as time function writing:

$$L_w(q(t)) = F(q(t)) \tag{18}$$

where F(.) represents the direct kinematic in (11).

In order to simplify the calculus in this paper, we can consider the following hypothesis.

Hypothesis 2: The structure is cylindrical D1=D2.

Under this hypothesis, the relation (11) remains valid, and

$$R_n = R - \Delta R \cdot \cos(\alpha_n) \tag{19}$$

Considering  $t_0=0$  and  $t_f = \delta t$  given, we can calculate the initial and final wires positions.

$$L^{0}_{w} = F(q(t_0))$$
(20)

$$L^{f}_{w} = F(q(\delta t)) \tag{21}$$

which is a column vector of three elements. From this we can compute the constant speed vector C.

$$C = \frac{L^{f}{}_{w} - L^{0}{}_{w}}{\delta t}$$
(22)

We are going to consider a small movement during  $\delta t$ .

$$L_w(q(t+\delta t)) = F(q(t)) + J_F(q(t)) \cdot \dot{q}(t) \cdot \delta t$$
(23)

where  $J_F$  is the Jacobian of F and where:

$$J_F(q(t)) \cdot \dot{q}(t) = C \tag{24}$$

Considering (12) we can compute the average length of the wires, during the small step.

$$\sum_{i} L_{wi}(q(t+\delta t)) = \sum_{i} (L_{wi}q(t) + C_i\delta t)$$
(25)

where  $C_i$  is the i-th component of the column vector representing the constant wire's speed.

$$\sum_{i} L_{wi}q(t) + C_{i}\partial t = \theta(t) \cdot \Delta R_{i} \cdot \cos\alpha_{i}(t) + L$$
  
+  $L - L - \theta(t + \partial t) \cdot \Delta R_{i} \cdot \cos\alpha_{i}(t + \partial t)$  (26)

Regrouping  $\theta(t + \delta t)$  and  $\theta(t)$  and using (12) we obtain:

$$\sum_{i} L_{wi}(q(t+\delta t)) = L$$
(27)

So during this movement there is no compression and no induced vibration (Boccolato *et al.*, 2009). If the average of the wires is different from L in fact, the solution we have is not the wanted solution. A "wrong" solution might introduce some structure twisting and during the movement them can became a rotational vibration of the structure. The proof can be extended to the tronconic structure with  $D_l \neq D_2$ .

Besides, in (27) there is no more dependence of L to  $\delta t$ . This means that, if the starting and the final points are solution of (18), moving with constant speed through the two points, we don't compress the structure. In the state space, for example, choosing a constant speed of the wire means move ( $\alpha$ ,  $\theta$  of the first segment as showed in (28).

$$\begin{cases} \dot{\alpha} = \frac{C_1 \cdot \sin(\alpha) \cdot \theta}{\Delta R \cdot (\sin^2(\alpha)) \cdot \theta^2 \cdot \cos^2(\alpha))} \\ \dot{\theta} = -\frac{C_1 \cdot \cos(\alpha)}{\Delta R \cdot (\sin^2(\alpha)) \cdot \theta^2 \cdot \cos^2(\alpha))} \end{cases}$$
(28)

This can be obtained by solving (24) for q(t):

$$J_F^{-1}(q(t)) \cdot C = \dot{q}(t)$$
(29)

#### **5 SIMULATION ON TENTACLE ARM**

#### 5.1 Motion control simulation

The result of the simulation of the control wires lengths variation when  $\theta$ =45° and  $\alpha$  goes from 0° to 360° as shown in Fig.7.



Fig 7. Variation of the wires when  $\alpha$  changes from 0° to 360°.



Fig. 8. Time evolution of the wire lengths without delays

The next experiment shows the influence of delays in the movement of the control wires on the virtual compression of the central bone. It is assumed that the segment moves from the initial position given by  $\theta_i=0^\circ$  and  $\alpha_i=0^\circ$  to the final position  $\theta_j=45^\circ$  and  $\alpha_i=45^\circ$  on the path that satisfies (30):

$$\dot{L}_{w1} = \frac{L_{w1}^{f} - L_{w1}^{01}}{t_{f} - t_{0}} = const$$

$$\dot{L}_{w2} = \frac{L_{w2}^{f} - L_{w2}^{0}}{t_{f} - t_{0}} = const$$

$$\dot{L}_{w3} = \frac{L_{w3}^{f} - L_{w3}^{0}}{t_{f} - t_{0}} = const$$
(30)

where and are the lengths of the control wires that correspond to the initial and final positions of the segment.

The simulation result from Fig. 8 shows that the length of the central bone remains constant as the segment moves from the initial to the final position, while control wires move with constant velocity. The simulation in Fig. 9 introduces a delay, such that the first control wire starts to move three seconds later than the other two. This can happen if one motors is looked or for some electric problems. As a result the central bone is compressed with approximately 2%. Compressing of 2% the central bone is not realistic, a part of the error go to really compress the central bone than the other part will twist the structure until a wrong solution will be found.



Fig. 9. Variation of the wires lengths when  $\alpha$  changes from 0° to 360 °

Wrong solution means a solution that cannot be described with only 2 DOF ( $\alpha$ ,  $\theta$ ).

## 5.2 Hardware architecture

The experimental mechanical structure consists in a three tentacle segment, resulting in a total length of more than one meter. Each segment's shape is controlled via three cables, actuated by three stepper motors, resulting in a 2 degree of freedom per element: 2 rotations around OX and OZ axis. The stepper motors are Berger Lahr, capable of 5000 steps per rotation. The rotation movement is converted in the translation needed for varying the length of the cables, by a roto-translation mechanism. The screw step is 1.25 mm, resulting in a 0.00025 mm displacement for each step made by a motor. The step motors are computer controlled, and the controllers Berger Lahr SD3 generate the command.

#### 5.3 Software architecture

The robot's control system consists in one computer running Windows XP and the Berger Lahr controllers interfaced by 3 modules Ad-link PCI8144. The main elements forming the software control architecture are the Visual C++ programming environment and the command functions included in the .DLL libraries provided by AD-Link. A program was developed in order to generate the displacement commands; this program computes the cable length variation in order to obtain a desired curvature and orientation, which determines the equivalent number of motor steps and transmits the command to the controllers, using the functions provided by Berger Lahr DLL's. This command scheme is represented in Fig. 10.

## 5.4 Problems and future work

In this paper we did not considered the errors introduced by the interaction between the segments. One of the future goals is to decouple the system mechanically or, if it is possible, in the state space. We designed and implemented a solution for a closed loop feedback through a machine vision system. This system will be used in order to determinate the real shape of the tentacle and his behavior.



Fig. 10. Software arhitecture

## 6. CONCLUSION

In this paper we presented the experimental tentacle structure designed and implemented at the Department of Mechatronics, University of Craiova, Romania. We focused our attention only on the actuation system of this robot. We have proposed a direct kinematical model and a moving algorithm. In particular, we considered a control algorithm for moving the robot from an initial position to a desired position implementing the kinematic of the structure. Further we developed some simulation tests to understand how synchronization's error, occurred while we move the robots, it will affect the desired shape. All the movement and the algorithm has been tested on the real robot after the simulation. As was considered from the beginning, the solution for determining the real shape and behavior of the tentacle is a machine vision system. This system is designed and implemented, but was not presented in this paper.

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