Nonlinear Identification of a Rotary Flexible Joint Experiment

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Abstract: Most systems that are encountered in practice are subjected to various uncertainties such as nonlinearities, actuator faults parameter changes etc. Modeling and identification of electro-mechanical systems constitute an essential stage in practical control design and applications. For a electro-mechanical rotational system, the nonlinearities, like Coulomb friction and dead zone, significantly influence the system operation when the rotation changes direction. The paper presents the black-box nonlinear identification of a rotational flexible joint setup. The nonlinear model for the system is obtained based on the discrete second order nonlinear Volterra model. Off-line identification of the nonlinear system model is performed using the least mean square algorithm. Results of the real time experiments are graphically and numerically presented, and the advantages of the nonlinear identification approach are revealed.

Keywords: Adaptive identification, Volterra model, Nonlinear systems, Flexible joint.

1. INTRODUCTION

Modelling and control problems of flexible structures have been extensively studied by many researchers (Luo 1993), (Matsuno, Murachi, and Sakawa 1994). These problems have arisen in the area of flexible space structures as well as in the area of lightweight flexible robot arms. Identification of electro-mechanical systems constitutes an essential stage in practical control design and applications. Controllers commanding systems that operate at varying conditions or re-quire high precision operation raise the need for a nonlinear approach in modelling and identification. Most electro-mechanical systems used in industry are composed of masses moving under the action of position and velocity dependent forces. These forces exhibit nonlinear behaviour in certain regions of operation (Pearson and Pottmann, 2000). For rotational systems, the nonlinearities, like Coulomb friction and dead zone, significantly influence the system operation when the rotation changes direction.

Growing needs for advanced and precise robot manipulators in space industry and mechanically flexible constructions result in new and complicated problems of modelling, identification and control of flexible structures, i.e. flexible beams, robot arms, etc. Dealing with flexible systems one is faced with inherent infinite dimensionality of the systems, light damping, nonlinearities influence of variable environment etc. One of the most important factors is to establish a suitable mathematical model of the system to make analysis as realistic as possible. Identification of nonlinear systems is of considerable interest in many engineering and science applications. Volterra series provide a general representation of nonlinear systems and have been applied in system identification.

This paper deals with the black-box nonlinear identification of a rotational flexible link experiment. The real mathematical models of these systems are very complicated, so for control purpose simplified models are typically used. In general, these models are derived using Lagrange's energy equations. In order to obtain useful models for control design, approximations of these models can be derived, therefore identification procedures are needed.

Some nonlinear models are obtained from first principles. Linearization is usually only possible around a specific operating point. But if the nonlinear system is used over the entire operating range the only alternative is, therefore, to model the system itself as a nonlinear model and estimate the parameters of this model on the basis of the measured input/output data. Probably the best known class of nonlinear systems that possess moving average representations is the Volterra model (Boyd and Chua 1985). Nonlinear models are frequently extracted using Volterra series and Wiener kernels because the model form does not need to be known a priori. Volterra models are very useful for signal and system representation due to their general nonlinear structure and their property of linearity with respect to their parameters, the kernel coefficients. Volterra series theory is the extension of the convolution integral to higher orders with the corresponding concepts of higher dimension impulse response functions and kernel transforms. For numerical treatment, the discrete version of the Volterra series expansion is more appropri-ate. In this paper one used a nonlinear least-mean squares (LMS) algorithm which is based on discrete second-order Volterra model. This nonlinear LMS method can be seen as an extension of the linear LMS algorithm. The merit of this ap-proach is that it keeps most of the of the linear LMS properties but still has a reasonably good convergence rate (Soderstrom and Stoica 1989). The proposed method is used to identify the flexible link setup. The input signal was generated by passing a zero-mean white Gaussian noise through a linear filter. First one performed a complete orthogonalization procedure for truncated Volterra series.

This allows us to use the LMS adaptive linear filtering algorithm to calculate all the coefficients recursively. This paper is organized as follows. The experimental setup that will be identified is described in Section 2. The discrete second order Volterra model is derived in Section 3. Section IV details the approach used to identify the nonlinear model using the least mean square algorithm. The experimental results are presented in Section 5 followed by some conclusions.

2. QUANSER ROTARY FLEXIBLE JOINT EXPERIMENT

The Quanser experimental set-up contains the following components (Quanser Consulting Inc. 1998): Quanser Universal Power Module UPM 2405/1503; Quanser MultiQ PCI data acquisition board; Quanser – Rotary Flexible Joint Module; Quanser SRV02-E servo-plant; PC equipped with Matlab/Simulink and WinCon software.

WinCon™ is a real-time Windows 98/NT/2000/XP applica-tion. It allows running code generated from a Simulink diagram in real-time on the same PC (also known as local PC) or on a remote PC. Data from the real-time running code may be plotted on-line in WinCon Scopes and model parameters may be changed on the fly through WinCon Control Panels as well as Simulink. The automatically generated real-time code constitutes a stand-alone controller (i.e. independent from Simulink) and can be saved in WinCon Projects together with its corresponding user-configured scopes and control panels.

The rotary motion experiments are based on the Rotary Servo Plant SRV02-E. It consists of a DC servomotor with built in gearbox whose ratio is 70 to 1. The output of the gear-box drives a potentiometer and an independent output shaft to which a load can be attached. The flexible link experiment consists of a mechanical and an electrical subsystem. The modelling of the mechanical subsystem consists in describing the tip deflection and the base rotation dynamics. The electrical sub-system involves modelling of DC servomotor that dynamically relates voltage to torque.

The rotary flexible joint consists of a rotary sensor mounted in a solid aluminium frame and is designed to mount to a Quanser rotary plant. The sensor shaft is aligned with the motor shaft. One end of a rigid link is mounted to the sensor shaft. The link rotation is counteracted by two extension springs anchored to the solid frame resulting in an instrumented flexible joint. The spring anchor points are adjustable to three locations to obtain various stiffness constants. Three types of springs are supplied with the system resulting in a total of 9 possible stiffness values. The link is also adjustable in length thus allowing for variations in inertia.

The equations of motion involving a rotary flexible joint imply modelling the rotational base and the flexible joint as rigid bodies. The major nonlinearities in the mechatronic systems of this kind are the Coulomb frictions, which are expressed as nonlinear functions of the rotational speeds, and the dead zone nonlinearity, which occurs between the overall system input and output. Although the Coulomb frictions are modelled as static nonlinear functions in our system, they introduce dynamic nonlinearities in the system input–output characteristics. This is a natural consequence of the fact that the static nonlinearities appear at the feedback loops of the two rotating masses.

Fig. 1. Quanser flexible joint experiment

System parameters are:

Armature Inductance	Lm	0.18mH
Armature Resistance	Rm	2.6 Ω
Motor torque constant	Km	0.00767Nm/A
Gear ratio	Kg	70
Load inertia	JLOAD	0.0059Kgm^2
Inertia (includes motor and gears)	JHUB	0.0021Kgm^2
Spring stiffness	KSTIFF	1.61N/m
Max applicable voltage	VMAX	5V

3. SECOND-ORDER VOLTERRA MODEL

Volterra models are very useful for signal and system representation due to their general nonlinear structure and their property of linearity with respect to their parameters, the kernel coefficients. However, when using Volterra models there are difficulties with a complexity problem that results from the very large number of parameters required by such models. Expanding the kernels on a generalized orthonormal basis allows to significantly reducing this parametric complexity.

Nonlinear models are frequently extracted using Volterra series and Wiener kernels because the model form does not need to be known a priori. Volterra series theory is the extension of the convolution integral to higher orders with the corresponding concepts of higher dimension impulse response functions and kernel transforms (Glentis, Koukoulas and Kalouptsidis 1999). For numerical treatment, the discrete version of the Volterra series expansion is more appropriate. The aim is to obtain values for the Volterra kernels over the desired time ranges by numerical deconvolution (Kibangou, Favier and Hassani 2003).

In this paper one develop a nonlinear least-mean squares (LMS) algorithm which is based on discrete second-order Volterra model. This nonlinear LMS method can be seen as an extension of the linear LMS algorithm. The merit of this approach is that it keeps most of the of the linear LMS properties but still has a reasonably good convergence rate. The proposed method is tested using computer simulated models. The input signal was generated by passing a zero-mean white Gaussian noise through a linear filter. First one performed a complete orthogonalization procedure for truncated Volterra series. This allows us to use the LMS adaptive linear filtering algorithm to calculate al the coefficients recursively. The method has been applied to identify a flexible link setup and a comparison with a linearized model was performed.

The zeroth-order Volterra model is just a constant defined as

$$Y_0(u_t) = h_0 \tag{1}$$

where u_t is the input signal and h_0 is a constant.

The first-order Volterra system is basically the same as the linear system. In other words, the linear system is a subclass of the Volterra system. Consider a general isolated linear system as shown in Fig. 1 where h_t^{N1} the represents the linear filter coefficients. The output y_t^{N1} can be expressed by input u_t as:

$$y_t^{N1} = u_t * h_t^{N1} = \sum_{k=0}^{\infty} h_t^{N1} u_{t-k} \tag{2}$$

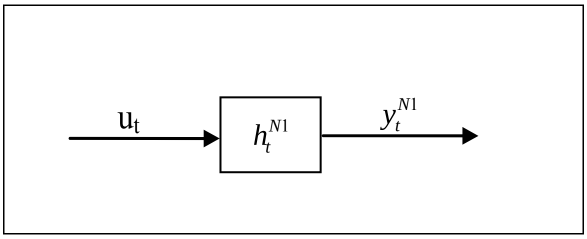

Fig. 2. First order linear system block diagram

In (2) the symbol * means linear convolution. If all the components in h_t^{N1} can be represented by some linear combination of orthonormal basis b_t, this means that the first-order Volterra kernel h_t^{N1} in equation (1) can be represented by:

$$h_t^{N1} = \sum_{m=0}^{\infty} a_m^{N1} b_t \tag{3}$$

where a_m^{N1} are some proper constants (Patra and Unbehauen 1993), (Kortmann and Unbehauen 1986). Note that $\{b_m, 0 \le m \le \infty\}$ is the set of orthonormal basis, which means:

$$\langle b_l, b_m \rangle = \delta_{l-m} \tag{4}$$

where $\langle\, , \rangle$ denotes the inner product and δ_{l-m} is the Dirac delta function. Substituting equation (4) in equation (3), one can define the first order Volterra functional as:

$$Y_1(u_t) = \sum_{k=0}^{\infty}\sum_{m=0}^{\infty} a_m^{N1} b_t u_{t-k} = \sum_{m=0}^{\infty} a_m^{N1}(b_t * u_t) \tag{5}$$

Note that equation (4) is the first-order homogeneous functional, which means that $Y_1(cu_t) = cY_1(u_t)$, where c is a constant. Equation (4) can be expressed by the block diagram shown in Fig. 2.
For the most general form of first-order Volterra system, one should include the DC term in equation (1), which can be expressed in terms of the Volterra functional $Y_0(u_t)$ and $Y_1(u_t)$ as

$$y_t = h_0 + u_t * h_t^{N1} = Y_0(u_t) + Y_1(u_t)\,. \tag{6}$$

From (6), one conclude that a general first-order Volterra system with DC term is one for which the response to a linear combination of inputs is the same as the linear combination of the response of each individual input.

The linear combination concept described above can be extended to the second-order case, which is one for which the response to a second-order Volterra system is a linear combination of the individual input signals. Consider the isolated second-order extension version of Fig. 3 shown in Fig. 4.

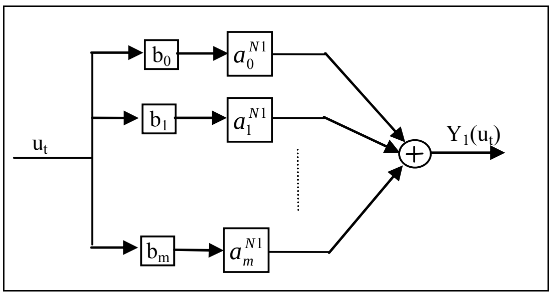

Fig. 3. First order Volterra model

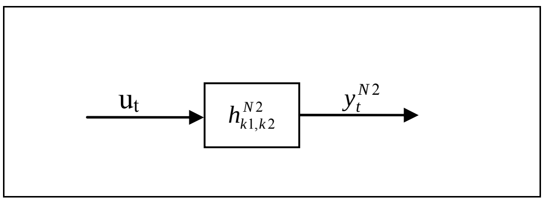

Fig. 4. Isolated second-order Volterra model block diagram

The response to the input ut in Fig. 3 is expressed by y_n^{N2} :

$$y_t^{N2} = \sum_{k1=0}^{\infty}\sum_{k2=0}^{\infty} h_{k1,k2}^{N2} u_{t-k1} u_{t-k2} \tag{7}$$

where $h_{k1,k2}^{N2}$ is defined as the second-order Volterra kernel. As in the literature [10], [11], [12], for simplicity and without loss of generality, one assume the kernel to be symmetric, which implies that $h_{k1,k2}^{N2} = h_{k2,k1}^{N2}$. If $h_{k1,k2}^{N2}$ can be expressed by the linear combination of orthonormal basis set b_t , then $h_{k1,k2}^{N2}$ can be written as

$$h_{k1,k2}^{N2} = \sum_{m1=0}^{\infty}\sum_{m2=0}^{\infty} a_{m1,m2}^{N2} b_{m1} b_{m2} \ . \tag{8}$$

Based on the description above, the general second-order Volterra system can be represented in terms of $Y_0(u_t)$, $Y_1(u_t)$ and $Y_2(u_t)$ which is

$$\begin{aligned} y_t &= h_0 + \sum_{k_1=0}^{\infty} h_{k1}^{N1} u_{t-k1} + \sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty} h_{k1,k2}^{N2} u_{t-k1} u_{t-k2} = \\ &= Y_0(u_t) + Y_1(u_t) + Y_2(u_t) \end{aligned} \tag{9}$$

For practical implementation a truncated series expansion is utilized that make use of last M input values. The truncated second order Volterra series expansion with memory M is

$$y_t = h_0 + \sum_{k_1=0}^{M-1} h_{k1}^{N1} u_{t-k1} + \sum_{k_1=0}^{M-1}\sum_{k_2=0}^{M-1} h_{k1,k2}^{N2} u_{t-k1} u_{t-k2} \tag{10}$$

4. IDENTIFICATION BASED ON VOLTERRA MODELS

Nonparametric methods of nonlinear system identification include those system representation methods as Volterrs kernel or Wiener kernel. Among these methods, Volterra kernel model is one of the most general model, since Volurra kernel model is considered to be an extension of impulse response model of linear case to nonlinear case. The Volterra series representation is an extension of linear system theory. This extension shows the highly complex nature of nonlinear filtering. In the following one used for identification a nonlinear least-mean squares LMS adaptive algorithm which is based on the discrete Volterra model. This nonlinear LMS adaptive method can be seen as an extension of the linear LMS algorithm. The merit of this approach is that it keeps most of the linear LMS properties but still has a reasonably good convergence rate. The performance analysis is also more tractable, which is seldom true for most nonlinear adaptive algorithms.

For identification one used an adaptive filter. Adaptive filters have been popular since the early 1960s after they were studied and developed in (Widrow and Stearns 1985). His development is based on the theory of Wiener filters for optimum linear estimation. There are other approaches to the development of adaptive filter algorithms, such as Kalman filters, least squares, etc. It has recently been shown that there is a close relationship between Kalman filters and recursive least squares adaptive filters (Syed and Mathews 1994), (Diniz 2002).

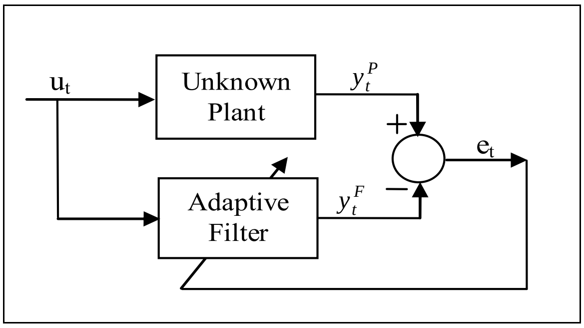

Fig. 5. System identification using adaptive filter

In the following, one develop the adaptive filter in the form of the least-mean square (LMS) algorithm. There are many variants of the LMS algorithm.

Fig. 6 illustrates the requirements of an adaptive filter, namely: a filter structure, a filter performance measure and an adaptive update algorithm. The input signal is u_t , the estimated response is y_t^F , the error signal is e_t and the real signal is y_t^P .

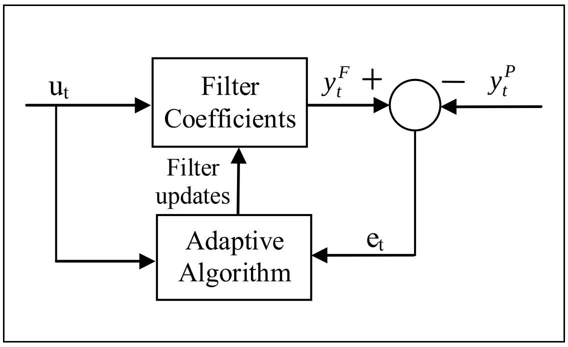

Fig. 6. Adaptive filter block diagram

The output of the adaptive filter y_t^F is compared with the plant response y_t^P . The error signal generated is used to adapt the filter parameters to make y_t^F more closely approach y_t^P or equivalently to make the error e_t approach zero. The minimization of a function of the error signal is used for the design. The choice of the performance function depends on the application. There are a variety of possible functions, but the most popular is the mean-square-error (MSE) with cost function:

$$J_t = | e_t^{\,2} | = e_t \cdot e_t^* \tag{11}$$

where $(.)^*$ means complex conjugate.

The steepest gradient descent method of minimization requires that one update the weight vector in the negative direction of the steepest gradient, according to the following formula:

$$H_{t+1} = H_t - \lambda \frac{\partial J}{\partial H} \tag{12}$$

This means one changes the weight vector in the direction of the negative gradient at each location on the performance surface at each instance of time, t.

For the second order Volterra model one have:

$$H_t = [h_0^{N1},\dots,h_{N-1}^{N1},\ h_{0,0}^{N2}, h_{0,1}^{N2},\dots,h_{N-1,N-1}^{N2}]^T \tag{13}$$

The regresors vector is

$$U_t = [u_t,\dots,u_{t-N+1},\ u_t^2, u_t u_{t-1},\dots,\ u_{t-N+1}^2]^T \tag{14}$$

Linear and quadratic coefficients are updated separately by minimizing the instantaneous square of the error $J_t = e_t^2$ where $e_t = y_t^P - y_t^F$. This results in the update equations:

$$h_{m1;t+1}^{N1} = h_{m1;t}^{N1} + \lambda_1 e_t u_{t-m_1} \tag{15}$$

$$h_{m1,m2;t+1}^{N2} = h_{m1,m2;t}^{N2} + \lambda_2 e_t u_{t-m1} u_{t-m2} \tag{16}$$

where λ_1 and λ_2 are step sizes used to control speed convergence and ensure stability of the filter.

Using the weight vector notation, H_t, one can combine the two update equations into one as the coefficient update equation

$$\begin{aligned} e_t &= y_t^P - H_t U_t \\ H_{t+1} &= H_t + \lambda U_t e_t \end{aligned} \tag{17}$$

It is evident that this procedure involves increasingly complex kernel expressions as the order of nonlinearity increases, and its practical efficacy is contingent upon our ability to obtain accurate kernel estimates from input-output data.

5. EXPERIMENTAL RESULTS

To obtain the nonlinear model of flexible joint setup, one considers the case of second-order Volterra filter with memory M=5, the output of which can be represented as in (10). The input signal used for identification was generated by passing a zero-mean white Gaussian noise through a linear filter. The variance of the input was chosen to be equal to 1. The input of the system is the supply voltage of DC motor and the output is the flexible arm deflection measured using a strain gage. The input and output signals used for identification are presented in Fig. 7 and Fig. 8. In Fig. 9 the convergence rate of a quadratic coefficient of Volterra kernels is presented. One can see that after 7000 iterations the LMS algorithm converge towards the solution. The simulated response of the identified nonlinear Volterra model to a impulse input of width equal to 1 sec and amplitude 2 Volts is presented in Fig. 9. The simulated response to the same input of a linearized model of the flexible joint experiment described in (Quanser Consulting Inc. 1998) is presented in Fig. 11.

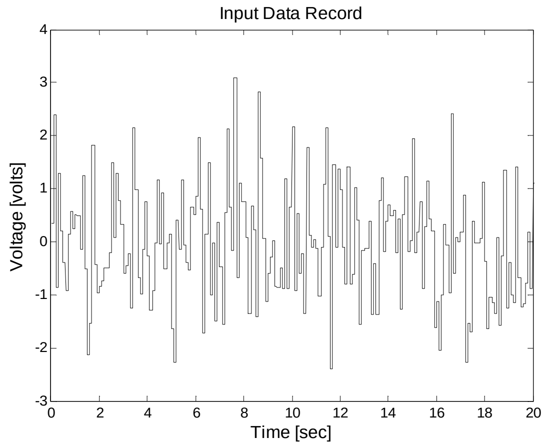

Fig. 7. Input signal (voltage)

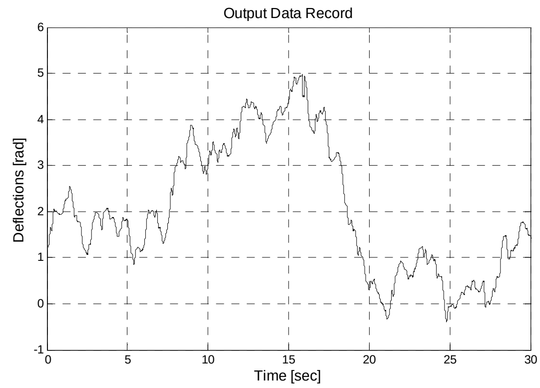

Fig. 8. Output signal (deflection)

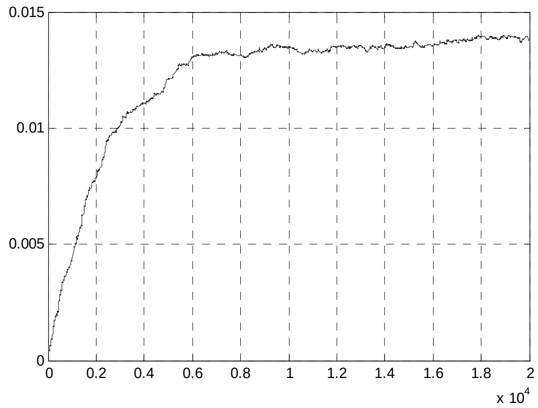

Fig. 9. Quadratic coefficient evolution

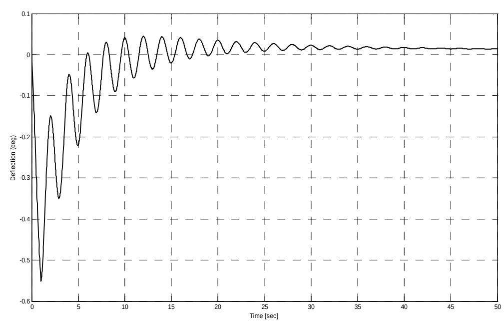

Fig. 10. Simulated response of nonlinear Volterra model

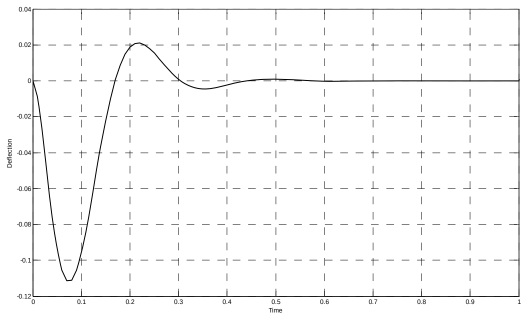

Fig. 11. Simulated response of linear model

6. CONCLUSIONS

In this paper, a black-box nonlinear identification of a rotational flexible joint setup has been presented. The nonlinear model for the system was obtained based on the truncated discrete second order nonlinear Volterra model. For numerical treatment, the discrete version of the Volterra series expansion is more appropriate. In this paper one used a nonlinear least-mean squares (LMS) algorithm which is based on discrete second-order Volterra model. This nonlinear LMS method can be seen as an extension of the linear LMS algorithm. The input signal was generated by passing a zero-mean white Gaussian noise through a linear filter. Results of the real time experiments are graphically presented, and the advantages of the nonlinear identification approach are revealed by comparing the impulse response of the nonlinear model obtained by identification and the impulse of the linearized model around a specific operating point. For identification one used an adaptive filter in the form of the least-mean square algorithm. The chosen performance function was the mean-square-error.

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