Thematic Session on Algebraic and Symbolic Methods for Mathematical Systems Theory

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Ever since the pioneering work of Kalman, algebraic methods have played a prominent role in mathematical systems theory, in particular, for analyzing the structural properties of a dynamical system, and for exploiting the resulting insights for the design of controllers. The theory has been greatly advanced by Fliess, Malgrange, Oberst, Pommaret, Sato, and many others. Nowadays, the development and use of specialized computer algebra packages makes this theory accessible to computational and experimental approaches. Many new and interesting results have been obtained by applying symbolic computation (possibly combined with numerical methods) in systems and control theory. For instance, structural system properties often have module-theoretic counterparts which can be made explicit using Gröbner basis techniques over certain skew polynomial rings. The goal of this thematic session is to present recent contributions in this field to the IFAC community through original and new results. The cross-fertilization between mathematical systems theory, algebraic tools, and symbolic computation will be highlighted for a wide range of functional equation types and system classes: linear and nonlinear, discrete and continuous, one-dimensional and multi-dimensional (1D and nD). Below, we briefly sketch some of the contributions we expect.

1. *Collision-free dynamical systems*, by E. Zerz (Aachen)

   Abstract: Collision-freeness is an important structural property of particle dynamics. A nonlinear ODE system $\dot{x}(t) = f(x(t))$ is called collision-free if the solution to the initial value problem with $x(0) = x_0$ has distinct components for all t whenever the initial state x_0 has distinct components. Both the general case of a C^1-function f and the special case of a polynomial (or even linear) function f will be addressed.

2. *Towards a geometric theory for nD behaviors: conditioned invariance and detectability subspaces,* by R. Pereira (Aveiro) and P. Rocha (Porto)

   Abstract: In this paper we study the notions of conditioned invariance for nD systems in the framework of the behavioral approach. This is inspired, but not completely equivalent to the one proposed in the classic geometric theory for 1D state space systems. The same happens with the concept of detectability sub-behavior, which will also be considered in our study.

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3. *New conditions for the intersection between algebraic curves and polydisks*, by Y. Bouzidi (Inria Lille)

   Abstract: One important question related to the stabilisation of multidimensionnal LTI discrete systems is to decide efficiently whether a polynomial system

   $$S := \{f_1(z_1, \ldots, z_n), \ldots, f_m(z_1, \ldots, z_n)\} \subset \mathbb{Q}[z_1, \ldots, z_n]$$

   has complex zeros that lie in the unit polydisk

   $$\overline{\mathbb{U}}^n = \{z = (z_1, \ldots, z_n) \in \mathbb{C}^n \mid |z_1| \leq 1, \ldots, |z_n| \leq 1\}.$$

   While in Strintzis, "Tests of stability of multidimensional filters", *IEEE Transactions on Circuits and Systems*, 24(8):432–437, 1977, simple conditions have been derived for the case of one polynomial ($m = 1$) which corresponds to an hypersurface, in this work, we propose an extension of this result to the case of $n-1$ polynomials ($m = n-1$) in complete intersection which corresponds to algebraic curves. As a consequence, new conditions for the intersection between algebraic curves and polydisk have been obtained, and an efficient algorithm based on computer algebra techniques has been developed for testing these conditions.

4. *On the computation of stabilizing controllers of multidimensional systems*, by Y. Bouzidi (Inria Lille), T. Cluzeau (Limoges), and A. Quadrat (Inria Paris)

   Abstract: In this paper, we consider the open problem consisting in the computation of stabilizing controllers of an internally stabilizable MIMO multidimensional system. Based on homological algebra and the so-called Polydisk Nullstellensatz, we propose a general method towards the explicit computation of stabilizing controllers. We show how the homological algebra methods over the ring of structural stable SISO multidimensional transfer functions can be made algorithmic based on standard Gröbner basis techniques over polynomial rings. The problem of computing stabilizing controllers is then reduced to the problem of obtaining an effective version of the Polydisk Nullstellensatz which, apart from a few cases, stays open and will be studied in forthcoming publications.

5. *Certified algebraic tests for the stability of fractional systems*, by Y. Bouzidi (Inria Lille) and T. Cluzeau (Limoges)

   Abstract: Testing the stability of a fractional system amounts to testing the existence of zeros of polynomials in some specific regions of the complex plane. In this paper, we propose a new algebraic approach to study the existence of complex zeros of univariate (resp. bivariate polynomials) in regions of the complex space defined as

   $$\{z \in \mathbb{C} \mid \arg(z) \in [-(\pi/2)\,\alpha, (\pi/2)\,\alpha]\},$$

   where $\alpha \in \mathbb{Q}$ (resp. $\{(z_1, z_2) \in \mathbb{C}^2 \mid \arg(z_i) \in [-(\pi/2)\,\alpha_i, (\pi/2)\,\alpha_i],\ i = 1, 2\}$, where $\alpha_1, \alpha_2 \in \mathbb{Q}$). The idea of our approach consists in using the algebraic representation of some specific sine and cosine values in order to reduce the problem to the computation of the sign of a univariate (resp. bivariate) polynomial at some algebraic number. This allows to test efficiently the stability of fractional systems in the so-called *commensurate* and *bi-commensurate* cases. We propose an implementation that validates our approach.

6. *A generalization of Fiagbedzi-Pearson's transformation for linear differential time-delay systems*, by A. Quadrat (Inria Paris)

   Abstract: In this paper, we show how an algebraic analysis approach to linear differential time-delay systems based on a ring of integro-differential time-delay operators can be used to generalized the so-called *Fiagbedzi-Pearson's transformation* which maps the trajectories of a linear differential systems with delayed state and delayed input to trajectories of a linear differential system without delays. As a particular case, we find again Artstein's transformation for linear differential systems with a delayed input.

7. *Certified symbolic-numeric computation of the H_∞-norm for LTI systems*, by Y. Grace (Inria Paris), Y. Bouzidi (Inria Lille), A. Poteaux (Lille), and A. Quadrat (Inria Paris)

   Abstract: The computation of H_∞-norm for linear time-invariant (LTI) systems plays a fundamental role in H_∞ control theory. This is usually done by means of a bisection method. In this paper, based on an approach developed by Kanno and Smith (*Validated numerical computation of the L_∞-norm for linear dynamical systems*, Journal of Symbolic Computation, 41 (2006), 697–707), we show how symbolic-numeric methods, developed by the computer algebra community, can be used to both certify and improve the efficiency of the computation of the H_∞-norm. Finally, we shall shortly study the case of LTI systems with parameters based on computer algebra methods for polynomial systems with parameters.

8. *Some algebraic aspects of signal demodulation*, by E. Hubert (Safran Tech), A. Barrau (Safran Tech), Y. Bouzidi (Inria Lille), and A. Quadrat (Inria Paris)

   Abstract: In this paper we introduce a general class of problems originating from gearbox vibration analysis. Building on a previous work where demodulation has been formulated as a matrix approximation problem, we are interested in the specific case applicable to amplitude and phase demodulation. This problem can be rewritten as a polynomial system with parameters. Based on algebraic methods, we shall focus on the characterization of the degree of generality of this problem and propose resolution methods.